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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 Calculus, 2007

90635 Differentiate functions and use derivatives to solve problems

Credits: Six
2.00 pm Thursday 22 November 2007

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show any derivatives that you need to find when solving the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Differentiate functions and use derivatives to solve problems.	☐	Demonstrate knowledge of advanced concepts and techniques of differentiation and solve differentiation problems.	☐	Solve more complex differentiation problem(s).	☐
Overall Level of Performance	☐				

You are advised to spend 50 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

Differentiate the following functions.
You do not need to simplify your answers.

(a) $y = (2x - 7)^{\frac{1}{4}}$

(b) $y = 3e^{4x} + \ln(5x + 6)$

(c) $y = (x^2 - 1)\tan x$

QUESTION TWO

Assessor's use only

In Singapore, tourists can ride up to view the skyline in a hot air balloon.

The hot air balloon is attached by a cable to the ground and rises vertically.

The height of the hot air balloon above the ground can be modelled by the function:

$$h = 5\sqrt{t} + 15\ln(t+1) \qquad 0 \le t \le 180$$

where h is the height in metres above the ground and t is the time in seconds after the ride begins.

Find the vertical velocity of the balloon after 15 seconds.

Give any derivatives that you need to find when solving this problem.

QUESTION THREE

Assessor's use only

A curve has the equation $y = (x^2 - 3x)^3$.

Write the equation of the tangent to the curve at the point $(2,-8)$.

Give any derivatives that you need to find when solving this problem.

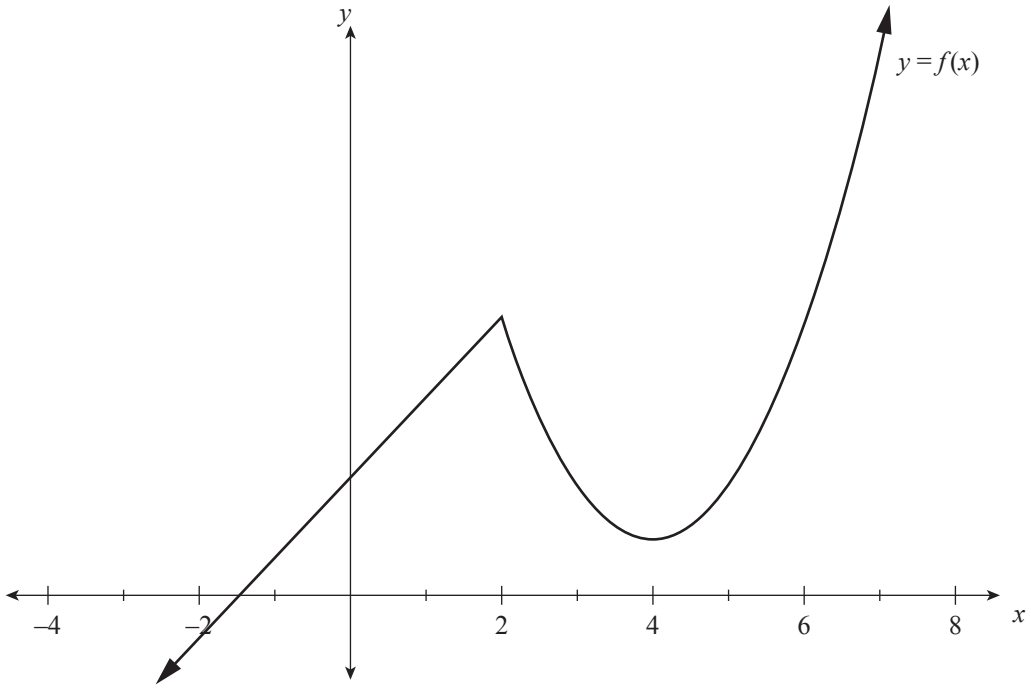
QUESTION FOUR

Find $\frac{dy}{dx}$ if $x^2 + 3x - 4y^2 = 5y$.

QUESTION FIVE

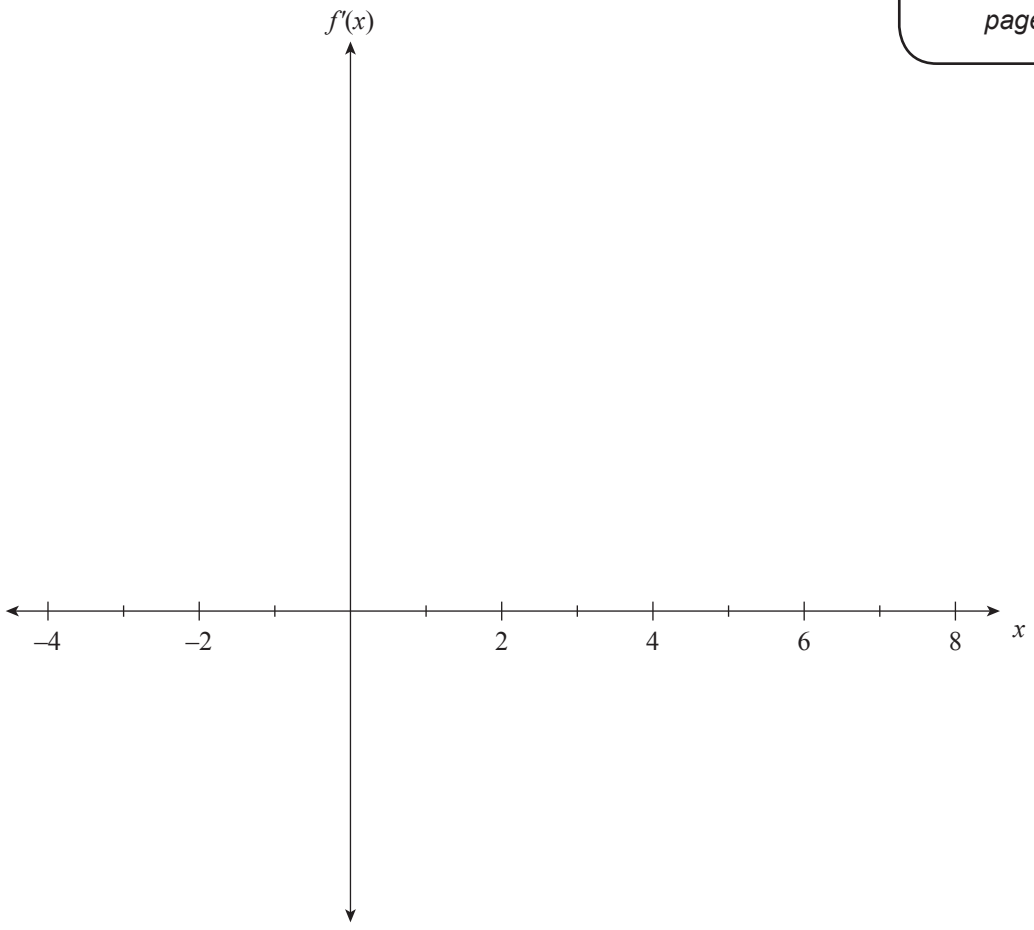
Assessor's use only

The graph below shows the function $y = f(x)$.



On the axes below, sketch the graph of $f'(x)$, the derived function of $f(x)$.

If you need to redraw this graph, use the grid on page 9.



QUESTION SIX

Assessor's use only

The Singapore hot air balloon descends vertically at a steady rate of 3 m s^{-1}.
Suzanne is standing on the ground 45 metres from the base position of the hot air balloon.
The angle of elevation, θ, of the hot air balloon from Suzanne decreases as the balloon descends.

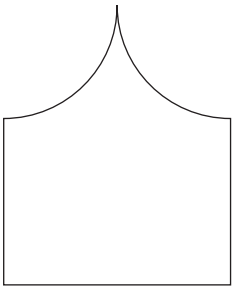
Calculate the rate of decrease of θ in radians per second when θ is $\frac{\pi}{4}$.

Give any derivatives you need to find when solving this problem.

QUESTION SEVEN

Assessor's use only

One of the hotel windows in Singapore is unusual.
Its shape is a rectangle that has two quarter-circles removed.
The radius, r, of both quarter-circles is half the width of the window.



Calculate the maximum area, A, of glass in such a window
if there is only 600 cm of frame available.

You may assume that $\frac{d^2A}{dr^2} < 0$.

Give any derivatives that you need to find when solving this problem.

QUESTION EIGHT

Assessor's use only

Find the values of t when the function defined by

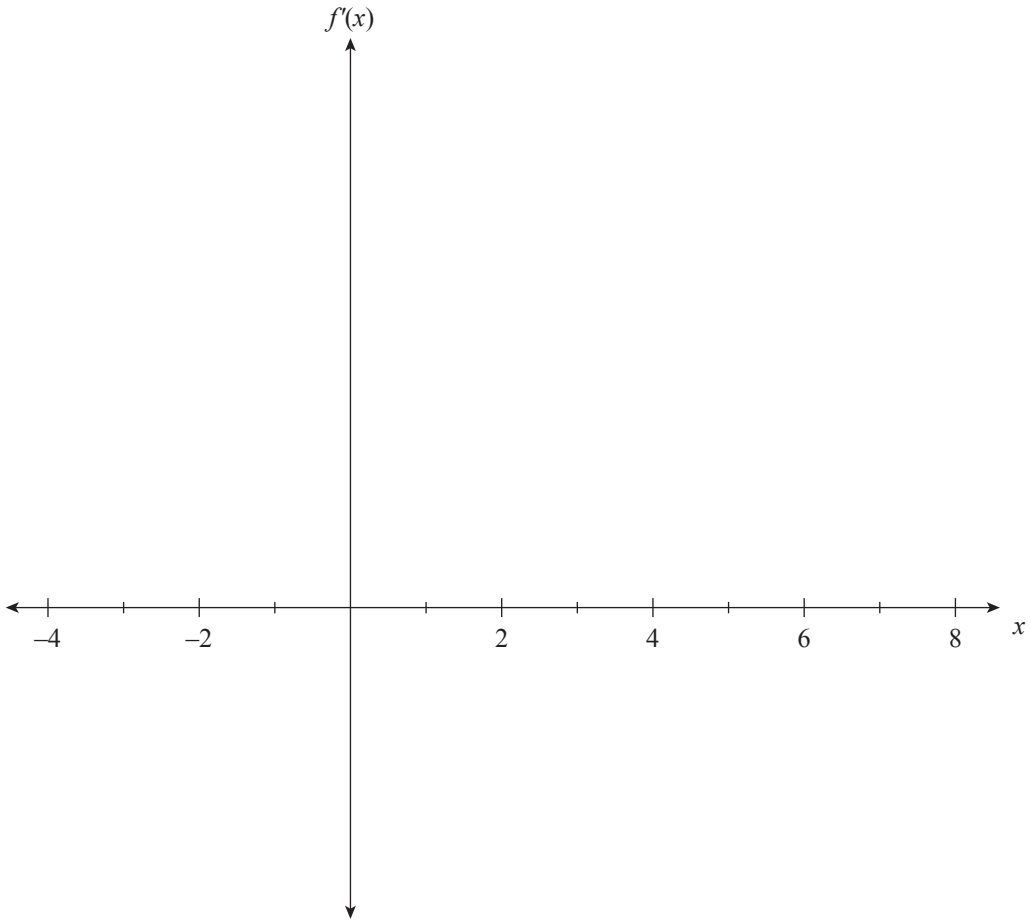
$x = 4t^3 + 1$ and $y = 3t^2 - 7t$ is concave up.

Give any derivatives you need to find when solving this problem.

If you need to redraw the graph from Question Five, draw it on the grid below.

Assessor's use only

You must cross out the graph that you do not want marked.



Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number	

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